BAYESIAN INVERSION OF THE FREE CORE RESONANCE

S. Rosat, N. Florsch and J. Hinderer (2007)

Please cite in your references:

- Florsch N. and J. Hinderer, 2000. Bayesian estimation of the free core nutation parameters from the analysis of precise tidal gravity data, *Phys. Earth Planet. Int.*, 117, 21-35.
- Rosat, S., Florsch, N., Hinderer, J. and Llubes, M., 2009. Estimation of the Free Core Nutation parameters from SG data: sensitivity study and comparative analysis using linearized Least-Squares and Bayesian methods, *J. of Geodyn*., 48, 331-339.

The FCN resonance formula is expressed by:

$$\tilde{\delta}_j = \tilde{\delta}_{ref} + \frac{\tilde{a}}{\sigma_j - \tilde{\sigma}_{nd}}$$

Here, the inversion of the observed gravimetric factors corrected for the ocean loading is performed to retrieve the following 3 FCN resonance parameters:

a^R (real part of the resonance strength); σ_{nd}^R (resonance eigenfrequency); $x = \log_{10}(Q)$.

The imaginary parts of $\tilde{a}$ and $\tilde{\delta}_{ref}$ are set to zero because of their small values and large uncertainties. Finally, the real and imaginary parts of the equations that we inverse are:

$$\begin{cases} \delta_j^R = \delta_{ref} + \dfrac{a^R(\sigma_j - \sigma_{nd}^R)}{(\sigma_j - \sigma_{nd}^R)^2 + \left(\dfrac{\sigma_{nd}^R 10^{-x}}{2}\right)^2} \\ \delta_j^I = \dfrac{\dfrac{a^R \sigma_{nd}^R 10^{-x}}{2}}{(\sigma_j - \sigma_{nd}^R)^2 + \left(\dfrac{\sigma_{nd}^R 10^{-x}}{2}\right)^2} \end{cases} \quad \text{with } \delta_{ref} = \frac{\delta_{O1}^R + \delta_{OO1}^R}{2}.$$

1/ Main program:

BAYES.FOR

Input:

* INI-FILE (e.g. "BAYES.INI"): File containing the options used in the program. Example of INI file:

|*INPUT FOR THE BAYESIAN INVERSION ROUTINE*
|*NUMBER OF STATIONS*
|*2*
|*FILE NAMES CONTAINING THE OBSERVED DELTA FACTORS*
|*FILE1.DAT*
|*FILE2.DAT*
|*INTERVALS OF SEARCH FOR THE 3 PARAMETERS AR,T AND X*
|*0.0005 0.0008*
|*380 540*
|*3 10*
|*NUMBER OF POINTS FOR THE GRID (<= 201)*
|*101*

* The input files FILE1.DAT and FILE2.DAT: contain the real and imaginary parts of the observed gravimetric factors corrected from the oceanic loading effect and their errors (so 4 columns) at the 1st and 2nd station. The last line of the file (line 10) corresponds to the reference gravimetric factor. By default we use 9 diurnal tidal waves:

Q1 O1 M1 P1 K1 PSI1 PHI1 J1 OO1

Output:

* "GGP_3" 1-column file containing the full probability density function values (3D density function);
* "X-T-.grd", "X-A-.grd" and "T-A-.grd" the joint density functions (2D marginal laws) normalized by 100.

2/ Routine programs:

delphi2drdi.f

Conversion of the gravimetric factors (amplitude, phase) outputting from ETERNA into real and imaginary amplitudes. This routine prepares the files used in BAYES.FOR (e.g. FILE1.DAT, FILE2.DAT) from the ETERNA analysis results. The oceanic loading correction can be performed with that routine too. In such a case, the loading file must be given as an input (e.g. “STRASBOURG.LOAD” using FES2004 ocean model) which contains 4 columns: *Amplitude cosine, amplitude sine, error for cosine, error for sine*

Example of input file: DELTA_ST9706.INI

```
**INPUT FOR PREPARATION OF DELTA FACTORS**
*OUTPUT FILE (BAYES_XXX)*
ST9706.DAT
*OCEAN MODEL CORRECTION? (0/1) *
*(0: NO CORRECTION - ALREADY DONE, IF 1: LOADING FILENAME) *
1
STRASBOURG.LOAD
1.15384
    0.501370 0.911390 Q1     59.0286   1.14765  0.00029  -0.3328   0.0143
    0.911391 0.947991 O1    308.3005   1.14895  0.00006   0.0591   0.0028
    0.947992 0.981854 M1     24.2339   1.15222  0.00067   0.2172   0.0331
    0.981855 0.998631 P1    143.4277   1.14971  0.00012   0.2369   0.0062
    1.001370 1.004107 K1    433.4073   1.13698  0.00004   0.2705   0.0021
    1.004108 1.006845 PSI1    3.3924   1.26374  0.00523   0.1750   0.2370
    1.006846 1.023622 PHI1    6.1720   1.17464  0.00291   0.4688   0.1420
    1.023623 1.057485 J1     24.2427   1.15678  0.00071   0.1744   0.0350
    1.057486 1.470243 OO1    13.2612   1.15654  0.00109   0.1364   0.0539
```

marginal.for

Routine to compute the 1 D-marginal laws from the 2D-joint density functions outputting from BAYES.FOR. The input file is one of X-Y-.grd where X and Y are the FCN resonance parameters x, σ_{nd}^{R} or a^{R}. The 1D-marginal law for X is computed by integrating on Y, and vice-versa.