



Challenges for forecasting based on accelerating rates of earthquakes at volcanoes and laboratory analogues

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SUMMARY

‘Mean-field’ models have been proposed as falsifiable hypotheses for the acceleration in earthquake rate and other geophysical parameters prior to laboratory rock failure and volcanic eruptions. Importantly, such models may permit forecasting failure or eruption time. However, in existing retrospective analyses it is common to find examples of inappropriate techniques for fitting these models to data. Here we test the two main competing hypotheses—exponential and power-law acceleration—using maximum likelihood techniques and an information criterion for model choice, based on a Poisson process with variable rate. For examples from the laboratory and Mt Etna, the power law is clearly the best model, both in terms of the fit and the resulting error structure, which is consistent with the Poisson approximation. For examples from Kilauea and Mauna Loa the results are less clear-cut and the confidence interval underestimates the number of outliers. Deviations from the models most likely reflect local interactions and/or non-stationary loading processes not captured by the mean-field approach. In addition, we use simulations to demonstrate an inherent problem with model preference, in that a power-law model will only be preferred if failure or eruption occurs close to the singularity. Although mean-field models may well provide valuable insight into the physical process responsible for precursory accelerations in earthquake rate, our findings highlight major difficulties that must be overcome to use such models for forecasting.

Key words: Time series analysis; Creep and deformation; Volcano seismology; Statistical seismology.

1 INTRODUCTION

Accelerations in the rate of volcano-tectonic (VT) earthquakes, and other geophysical proxies for deformation, are frequently reported before volcanic eruptions (Tokarev 1971; Voight 1988; Voight & Cornelius 1991; McGuire & Kilburn 1997; Kilburn & Voight 1998; Chastin & Main 2003; Collombet *et al.* 2003; Sparks 2003; Smith *et al.* 2007). Modelling such accelerations may determine the processes controlling the approach to eruption and allow forecasting of future activity. Using observations of deformation, Voight (1988) proposed and applied an empirical relation to describe accelerations preceding volcanic eruptions. As the eruption approaches, the acceleration in a geophysical signal Ω is related to its rate by

$$\frac{d^2\Omega}{dt^2} = K \left(\frac{d\Omega}{dt} \right)^\alpha, \quad (1)$$

where α and K are constants. Solutions to eq. 1 involving positive

acceleration take the form of either a power law ($\alpha \neq 1$) or an exponential ($\alpha = 1$) increase in the rate of precursory signals with time (Voight 1988; Kilburn & Voight 1998; Main 1999). Solutions with $\alpha > 1$ involve a singularity at a finite time, correspondingly to an instantaneously infinite rate, and frequently interpreted as the likely eruption onset. Although several different measurements have been used for Ω (Voight 1988; McGuire & Kilburn 1997), the number of VT earthquakes is most frequently employed, both because this is a reasonable proxy for the accumulation of brittle deformation in the volcanic edifice, and because the data sets are readily available for processing during a crisis.

Although it has been argued that robust accelerating rates of VT earthquakes prior to basaltic eruptions are only observed when many pre-eruptive sequences are averaged (Chastin & Main 2003; Collombet *et al.* 2003), several studies present convincing evidence for accelerating precursors before individual eruptions and intrusions (Vinciguerra 2002; Lengliné *et al.* 2008). Where they are observed, accelerating sequences before volcanic eruptions resemble those in the rate of acoustic emissions (AE) prior to failure in laboratory rock-physics experiments. Consequently, eq. 1 has been interpreted as generally reflecting the approach of bulk or mean-field

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properties to brittle failure (Voight 1989; Anifraní *et al.* 1995; Garcimartin *et al.* 1997; Guarino *et al.* 1998), with models proposed to explain its origin in terms of fracture growth and interaction under a constant or steadily increasing stress (Kilburn & Voight 1998; Main 2000; Kilburn 2003). Alternative mean-field models have been proposed to describe the approach to eruption on the basis of different stress regimes (e.g. Lengliné *et al.* 2008). However, these commonly also predict an exponential or power-law increase in the acceleration of VT earthquake rate.

Inherent uncertainty is involved when choosing between plausible models on the basis of their goodness-of-fit to observations, even when employing the most rigorous techniques. It is generally assumed that earthquake occurrence is a Poisson process whose mean rate varies in time and space (Marsan & Nalbant 2005) and whose rate uncertainties in relatively short time intervals should therefore be approximately Poisson distributed. That is, the variance of the random uncertainty about the mean rate in each time interval, $\sigma^2(d\Omega/dt)$, is equal to that mean rate $d\Omega/dt$. However, it is common to find analyses based on inappropriate model-fitting techniques such as (i) least-squares regression on discrete data and (ii) regression of any kind on cumulative data (Voight 1988; De la Cruz-Reyna & Reyes-Davila 2001; Lengliné *et al.* 2008). Model parameters and statistics derived from such analyses are invalid and potentially misleading (Vere-Jones *et al.* 2001; Hardebeck *et al.* 2008; Greenhough *et al.* 2009), regardless of the suitability of the models themselves. Consequently, there are clear implications for the accuracy of any subsequent forecasts.

In this paper we demonstrate how established techniques for model-fitting and preference can be applied to accelerations in volcanic earthquake and laboratory AE data. We discriminate retrospectively between power-law and exponential models for accelerations in event rate. We show that the Bayesian Information Criterion (BIC) is a useful pragmatic measure of model preference, using simulated earthquake sequences from both models. We then apply these techniques to illustrative examples from basaltic volcanoes and laboratory brittle-creep experiments. We highlight the difficulty in choosing between models unless the final few data points are available, and that in some accelerating earthquake rates prior to volcanic eruptions, there are systematic departures from the model with time. Our findings indicate that quantitative forecasting on the basis of mean-field models will be associated with significant challenges.

2 SIMULATED ACCELERATIONS TO FAILURE

Fig. 1(a) shows a simulated accelerating sequence of earthquakes whose mean rates are proportional to $[t_f - t]^{-p}$, where $p = 0.7$, with t_f , the failure time, fixed before fitting. Note that for this model t_f is defined by a singularity. We use the maximum likelihood code of Ogata (2006), to fit a power-law acceleration and an exponential acceleration of form $\exp(-\lambda|t_f - t|)$ to the observed event times (note, not to the cumulative event number); these are plotted in Fig. 1(b). As the simulated data consist of a series of discrete events, the observed rates will vary about the modelled mean rates according to Poisson distributions. Given most rates lie scattered within the 95 per cent confidence interval, the data might be considered consistent with both models; how then can we choose between them? On one hand, there are suggestions of systematic deviations from the wrong-model fits, which may be sufficient grounds for their rejection. On the other hand, if we decide these deviations

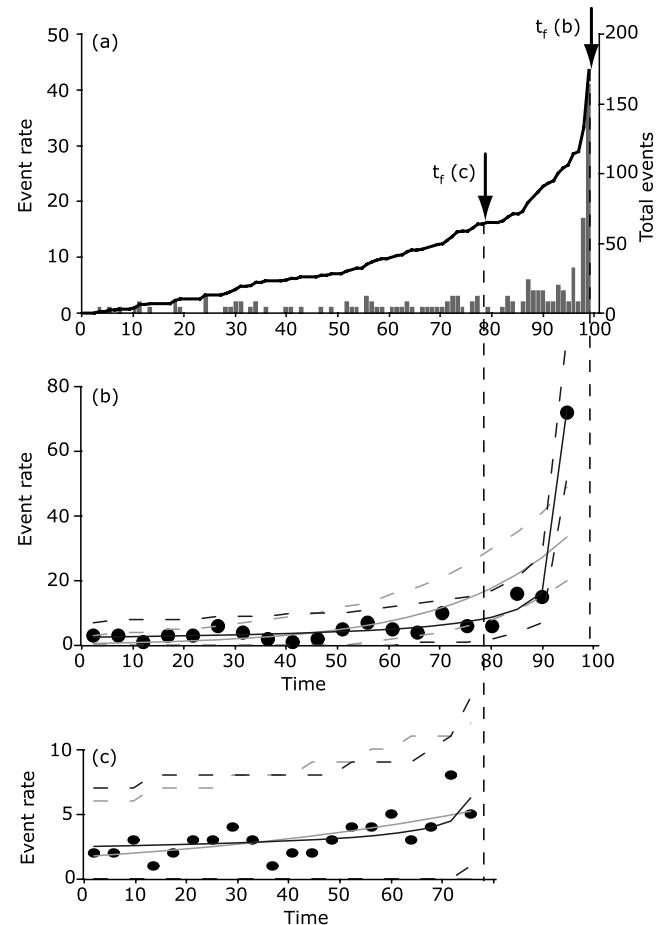


Figure 1. (a) Daily and cumulative numbers of events in a simulated earthquake sequence with an underlying power-law acceleration in mean rate with $p = 0.70$. (b) Maximum likelihood estimated power-law (solid dark grey; $p = 0.71$) and exponential (solid light grey; $\lambda = 0.04$), fits to event rates for entire sequence along with the 95 per cent confidence intervals for the observed rates (dashed). (c) As for (b) but for sequence truncated 20 days before power-law singularity ($p = 0.22$, $\lambda = 0.015$).

are insignificant and have no prior model preference, then the BIC provides a simple means of choosing the best model based on the likelihood of the observations, with a bias towards the model with fewer parameters. The BIC has been used widely for pragmatic decision making, as supporting (though not sufficient) evidence for a model; see Kass & Raftery (1995) for a comprehensive review.

The BIC is given by

$$\text{BIC} = -2\ln L + P\ln(n), \quad (2)$$

where L is the likelihood of the observations given the model, P is the number of free parameters and n is the number of observations. In making an inference, the preferred model is more likely to have the lower BIC. Where ΔBIC is the positive difference in BIC between the two models, the difference in likelihood is given by $\exp(\Delta\text{BIC}/2)$ in the case where both models have the same number of parameters. Attaching conventional confidence levels to BIC differences is difficult (Kass & Raftery 1995) though it is possible to estimate the expected number of false-positives via simulations (i.e. in how many simulations does the true model have a higher BIC), with the caveats that (i) fits to simulations need to be validated in terms of both visual scatter and parameter plausibility and (ii) simulations are unlikely to capture the complexity of the real system.

Note that we are not considering the parameter estimates *per se*, for which significant bias and uncertainty may exist for the small sample sizes routinely available. In addition, it is important to try different data selection criteria and initial estimates of parameters to assess the robustness of the results.

For the data in Fig. 1(b), we have BIC equal to -120 and -7 for the power-law and exponential models, respectively. Since both models have two parameters (exponent and multiplicative constant), the BIC difference is equal to twice the log of the ratio of the models' likelihoods. As expected, the lower BIC (-120) corresponds to the model used for simulation, and this difference is considered 'decisive' evidence in favour of that model (Raftery 1993). As a first-order guide to its reliability for this specific scenario, we simulate 500 exponential and 500 power-law accelerating sequences, each of 100 events, and each fitted by both models. The lower BIC identifies the correct model in 90 per cent of cases. This success rate is dependent on the total number of observations; the same simulations of 20 events returned the correct model in 70 per cent of cases and for 1000 events the success rate was 99 per cent.

Models predicting a power-law acceleration in the rate of earthquakes generally associate macroscopic failure or the onset of eruption with the singularity, where the rate theoretically tends to infinity. As no singularity is defined for an exponential model, such events must be associated with fulfilment of a critical strain or strain rate criterion. However, it is also possible that such a criterion may apply for the power-law model, with failure or eruption occurring before the singularity is reached; such a possibility has important implications for model-fitting and preference. To demonstrate this, Fig. 1(c) shows the same simulated power-law accelerating earthquake sequence as in Fig. 1(b), but with the hypothetical eruption time 20 d earlier. We now refit both power-law and exponential functions to this truncated power-law sequence, maintaining the assumption that the eruption time corresponds to the power-law singularity. Interestingly, in this scenario $\Delta\text{BIC} = 1$, indicating a marginal preference for the exponential model (1.6 times more likely), even though the

data were generated by a power law, with similar results obtained for multiple simulations.

3 EXAMPLES OF ACCELERATION TO FAILURE FROM NATURAL SYSTEMS

We now consider examples of accelerating rates of VT earthquakes from basaltic volcanoes Kilauea and Mauna Loa, Hawaii and Mt Etna, Sicily and AE data from laboratory brittle-creep experiments. Lengliné *et al.* (2008) present examples of accelerating VT earthquake sequences preceding the 1983 eruption at Kilauea, 1984 eruption at Mauna Loa, and fit both power-law and exponential models to cumulative earthquake numbers. However, they use an inappropriate statistic, R^2 derived from least-squares regression on cumulative data (see Section 1), to support a preference for the exponential model. Here we reanalyse these sequences, applying the same spatial, temporal and magnitude data selection criteria as Lengliné *et al.* (2008) to enable direct comparison. To perform the model fits, we again apply the maximum likelihood code of Ogata (2006) with a known failure time. We then validate the resultant fits by considering the rates, which should be randomly scattered within some confidence interval.

Figs 2(a) and (b) show the daily earthquake rate and total number of earthquakes as a function of time, and best-fit power-law and exponential models of the earthquake rates, preceding the 1983 eruption at Kilauea and the 1984 eruption at Mauna Loa. Due to the large number of observed rates outside the 95 per cent confidence intervals, many more than the expected one in 20, the data from Kilauea are not consistent with either model and hence derived statistics, and any forecasts that could be issued using them, are not valid. In contrast, the fluctuations in earthquake rates in the Mauna Loa sequence are consistent with the Poisson confidence intervals of both models. However, they are not randomly scattered along the sequence for the power-law model, instead lying

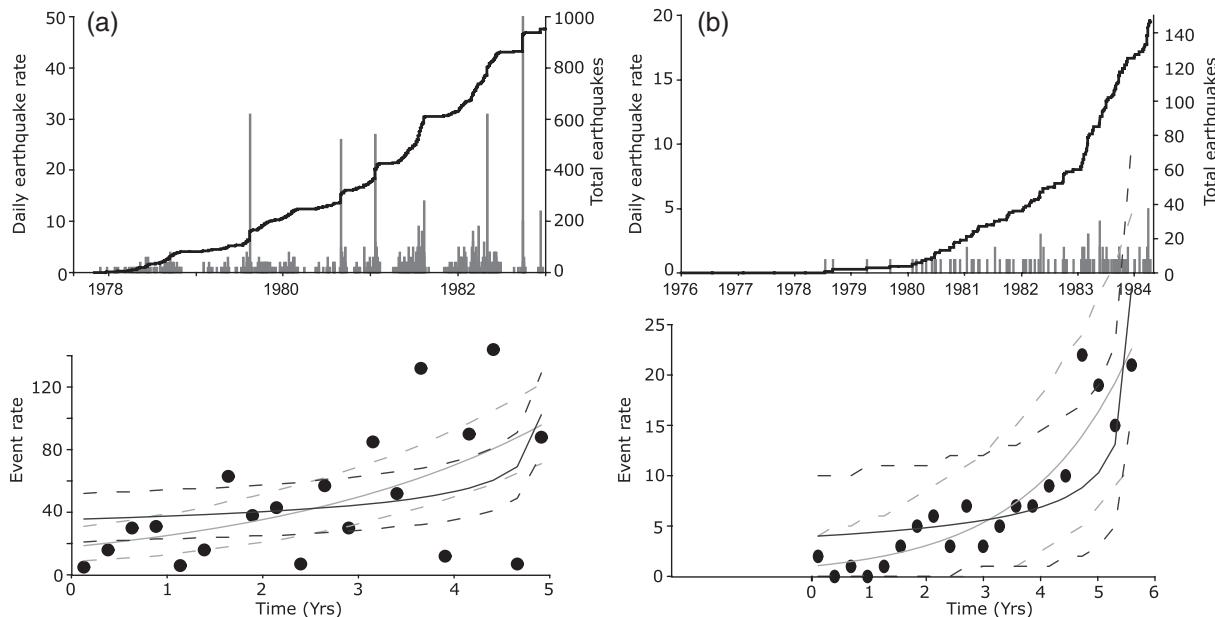


Figure 2. Accelerating rates of earthquakes preceding the (a) 1983 eruption at Kilauea and (b) 1984 eruption at Mauna Loa. Data have been selected according to the criteria of Lengliné *et al.* (2008). Top panel: daily and cumulative numbers of events. Bottom panel: rates, maximum likelihood power-law (solid dark grey) and exponential models (solid light grey) and 95 per cent confidence limits (dashed). Estimated model parameters are (a) $p = 0.26$, $\lambda = 0.001$ and (b) $p = 0.46$, $\lambda = 0.002$.

systematically below and above the fit, and hence the exponential model is preferred.

The two accelerating sequences in Figs 2(a) and (b) evolve over timescales in excess of 5 yr. The sequence from Kilauea covers a period during which many smaller eruptions and dyke intrusions occurred, themselves commonly associated with accelerating rates of earthquakes on shorter timescales (Klein *et al.* 1987). We now analyse two such accelerations preceding the 1981 August and 1982 June dyke intrusions (Figs 3a and b), along with accelerating rates of magnitude ≥ 2.5 earthquakes preceding the 1989 September eruption of Mt Etna (Vinciguerra 2002, Fig. 4). These sequences evolve over timescales between 120 and 500 d. In all three cases, the observed earthquake rates are consistent with expected random variation around both models. For the two examples for Kilauea, $\Delta\text{BIC} = 11$ and 14 in favour of the exponential with respective likelihood ratios of $\exp(\Delta\text{BIC}/2) \sim 10^2$ and 10^3 . For the example from Mt Etna, it is the power law that is preferred ($\Delta\text{BIC} = 21$, likelihood ratio $\sim 10^4$). However, in all three cases the earthquake rates show signs of small temporally correlated deviations above and below these models, suggesting the influence of additional factors not captured by a mean-field approach, such as material heterogeneities that affect bulk properties disproportionately.

Laboratory brittle-creep experiments have been proposed as an analogy for macroscopic failure preceding volcanic eruptions (Voight 1989; Main 2000). Like VT earthquakes, AEs are generated by brittle failure (Benson *et al.* 2008), consequently both phenomena are considered to reflect the progression of damage towards macroscopic failure. Here we analyse AE data from a previously published laboratory brittle-creep experiment to illustrate appropriate model-fitting and preference techniques in a potentially related scenario, but on a vastly smaller scale. Fig. 5 shows the number of AE events per minute and total number of AE events as a function of time recorded during a brittle-creep experiment on a sample of Darley Dale sandstone. The sample was held at a constant stress of 125 MPa (a high proportion of its short-term failure stress) until

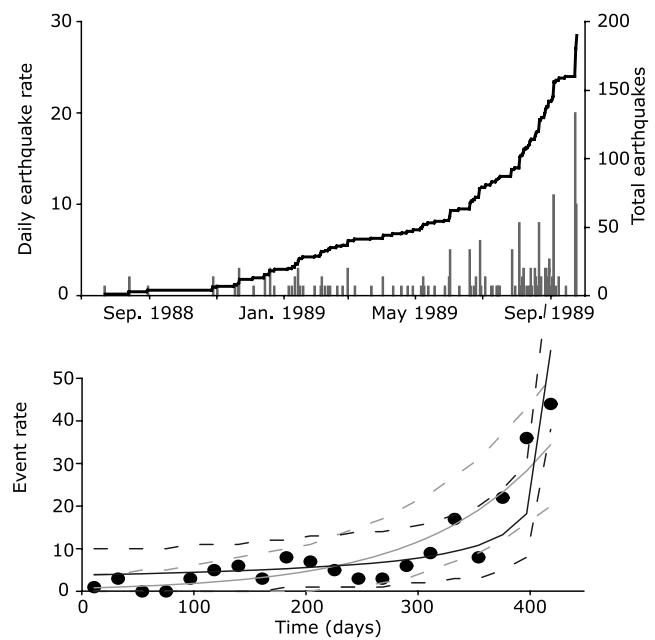


Figure 4. Accelerating rates of earthquakes preceding the 1989 September eruption at Mt Etna. Top panel: daily and cumulative numbers of events. Bottom panel: rates, maximum likelihood power-law (solid dark grey) and exponential models (solid light grey) and 95 per cent confidence limits (dashed). Estimated model parameters are $p = 0.6$, $\lambda = 0.009$.

acceleration to failure, after approximately 3600 min, promoted by stress corrosion reactions. During secondary creep the sample deformed at a steady-rate creep strain rate of $1.3 \times 10^{-8} \text{ s}^{-1}$. Details of the experiment are published in Heap *et al.* (2009). We take the final 800 min of the experiment which involve the acceleration to failure, and cut the AE catalogue according to a completeness magnitude of 1.5 log(energy) units. The power-law and exponential best-fit

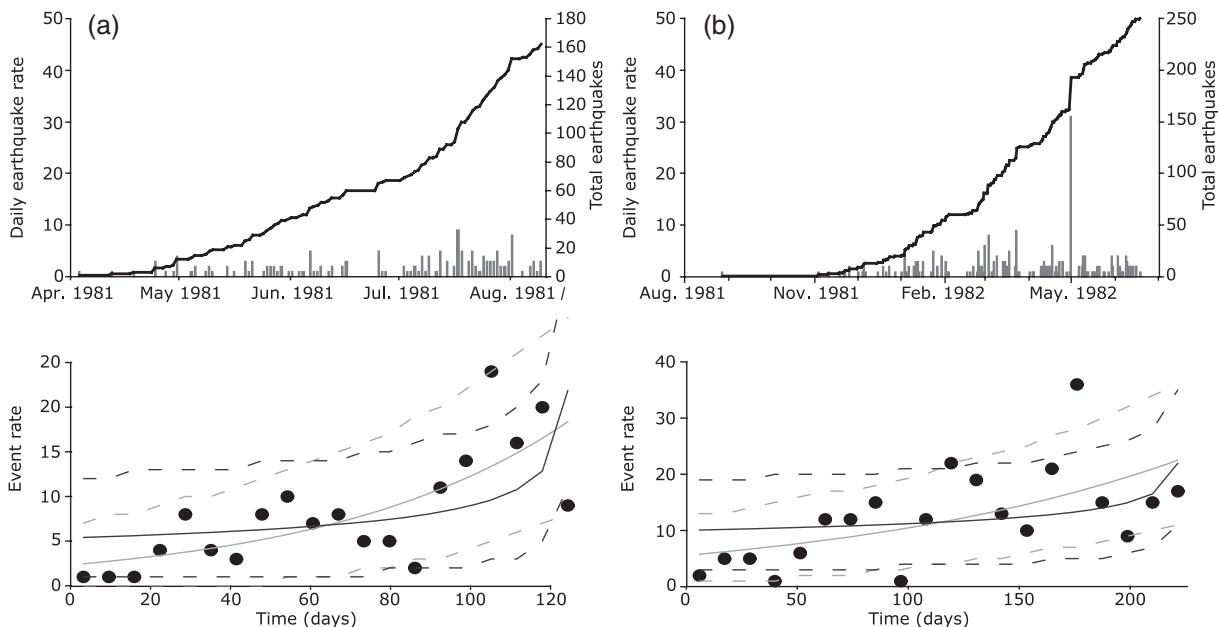


Figure 3. Accelerating rates of earthquakes preceding the (a) 1981 August and (b) 1982 June intrusions at Kilauea. Data have been selected according to the criteria of Lengliné *et al.* (2008). Top panel: daily and cumulative numbers of events. Bottom panel: rates, maximum likelihood power-law (solid dark grey) and exponential models (solid light grey) and 95 per cent confidence limits (dashed). Estimated model parameters are (a) $p = 0.33$, $\lambda = 0.016$ and (b) $p = 0.19$, $\lambda = 0.006$.

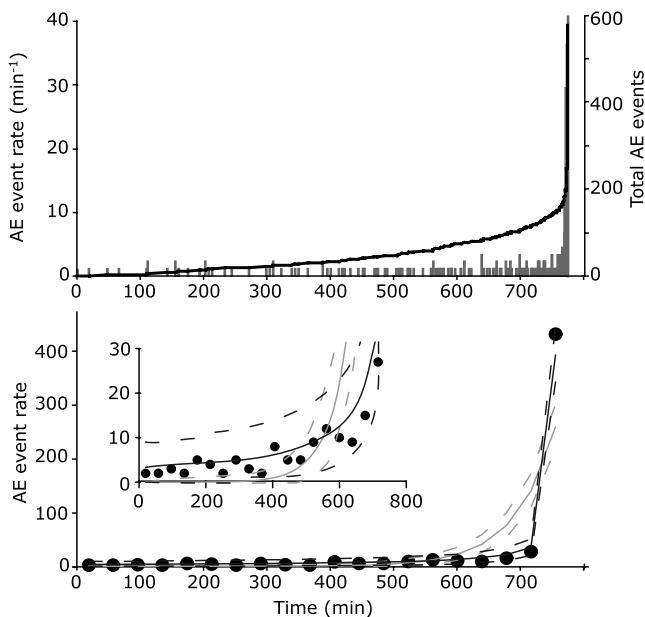


Figure 5. Accelerating rates of acoustic emissions preceding sample failure during a brittle-creep experiment (Heap *et al.* 2009). Top panel: daily and cumulative numbers of events. Bottom panel: rates, maximum likelihood power-law (solid dark grey) and exponential models (solid light grey) and 95 per cent confidence limits (dashed). Estimated model parameters are $p = 0.9$, $\lambda = 0.016$. Inset shows exaggerated y-axis for clarity.

models are again determined using the maximum likelihood code of Ogata (2006). The data show a rapid acceleration just before failure that is consistent only with the power-law model (with $p = 0.9$) and there are no obvious systematic fluctuations about this model on the scale of the bin widths.

4 DISCUSSION

We have demonstrated appropriate techniques for fitting and distinguishing between power-law and exponential models for accelerations in volcanic earthquake and laboratory AE data. Our intention is not to suggest one model is preferred in all cases. Indeed, on the basis of model validity and/or the BIC, although the power-law model is preferred for the examples from Mt Etna and the laboratory, the exponential model is preferred for the examples from Mauna Loa and the two intrusions at Kilauea. Analysis of a far more extensive set of sequences would be required to establish whether either model, or an alternative model altogether, is most likely in general. It is possible that such a data set does not yet exist.

Much of the difference between the power-law and exponential models appears close to the ends of the sequences. Even in the case of retrospective model evaluation, Fig. 1(c) demonstrates that a power-law acceleration can only be identified if eruption or sample failure approximately coincide with the power-law singularity. Consequently, it is possible that the volcanic sequences for which an exponential model is preferred are actually power-law accelerations, but with the onset of eruption determined by a critical strain or strain-rate criterion that is satisfied significantly before the power-law singularity. Such a criterion could correspond to a sufficient local reduction of the minimum principal stress to permit dyke injection, for example. Theoretical considerations also suggest that it may be possible for a sequence to evolve with time from an exponential to power law (McGuire & Kilburn 1997; Kilburn 2003). Such a transition will be difficult to confirm in a forecasting scenario

until close to the eruption unless there are strong prior constraints on parameters and timescales.

This observation raises a fundamental difficulty with forecasting on the basis of such models. An exponential model requires failure to occur before an effectively infinite earthquake rate is reached, but a power-law model with such a criterion will be indistinguishable from an exponential. Consequently, without prior constraints on parameter values, we suggest it is unwise to use data fitting to prefer an exponential model. Hence in a forecasting scenario, where analysis can only be carried out on the early part of the sequence, model preference based solely on the available data could be highly misleading. Reliable forecasting might be possible only where strong prior constraints exist on eruption criteria or parameter values.

In real data, model-fitting and preference will be further complicated by the existence of earthquake clustering. Given an underlying acceleration in the mean rate of earthquakes, earthquake triggering will lead to greater temporal clustering than in a Poisson process (Gardner & Knopoff 1974; Huc & Main 2003). The presence of clustering will only make the best model and parameters even more uncertain. Although there is little research on earthquake clustering in volcanic data sets, in future both retrospective analysis and prospective forecasting will require the use of declustering techniques or models which account for earthquake triggering (Helmstetter & Sornette 2003). In addition, earthquake data could be combined with additional independent sources of information, such as ground deformation or geochemical signals, to provide improved forecasting schemes.

In all the volcanic sequences, we identify systematic deviations from the simple power-law and exponential models. Volcanoes and laboratory rock samples are complex systems that comprise many interacting processes but whose bulk behaviour may be captured by comparatively simple models. Although certain processes may be dominant at small scales, at large scales these may be modelled as stochastic fluctuations around a deterministic trend. However, depending on the relative scales and amplitudes of the processes involved, certain processes may appear as systematic deviations from simple models. Therefore, earthquake rate data is insufficient to distinguish between models. Analysis of larger data sets than are currently available, and of other properties such as magnitude–frequency distributions and spatial and temporal clustering, may eventually allow this distinction to be made.

In the laboratory, relatively homogeneous rock samples are chosen with a grain size an order of magnitude smaller than the sample. This selection imposes a minimum relative scale difference between the bulk sample behaviour and that of individual grains, allowing the data to be modelled as a simple deterministic trend with stochastic (Poisson) fluctuations as seen in Fig. 5. However, the relative heterogeneity of volcanic settings introduces multiple processes on scales comparable to the whole system. Depending on the volcano and on the model considered, these may include failure of large material asperities, changes in regional stress conditions [e.g. due to flank instability, Brooks *et al.* (2006)] or changes in magma pressure due to small eruptions or intrusions. Such processes give rise to systematic deviations from simple model predictions for the bulk system and hence degrade the forecasting power.

5 CONCLUSIONS

Accelerating rates of earthquakes are frequently observed before volcanic events and failure in laboratory analogues. However, such sequences are routinely modelled using inappropriate techniques,

particularly least-squares regression on cumulative quantities. In this paper we show how volcanic and laboratory data sets can be analysed using (1) a maximum likelihood technique to fit power-law and exponential rate models to sequences of earthquake times, and (2) the BIC for model preference where the data are consistent with both. We find that for different volcanic and laboratory sequences the power-law or exponential models may be preferred, but that in volcanic data there frequently exist significant systematic deviations that invalidate these mean-field models. Consequently, even in retrospect, it is only possible to identify a power-law model if the culmination of the accelerating sequence is close to the singularity. The deviations from the model trends, coupled with the sensitivity of model preference to the final few points of the sequence, imply that the uncertainties in forecasts from exponential or power-law models are currently very difficult to quantify. Hence in a forecasting scenario, model preference and predicted eruption times are likely to only be reliable a short time in advance.

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